P23CCT22 - FINITE ELEMENT METHODS IN MECHANICAL DESIGN

Syllabus:

UNIT I FINITE ELEMENT ANALYSIS OF ONE DIMENSIONAL PROBLEMS	**No. of Periods: 9+3**
Historical Background – Weighted Residual Methods - Basic Concept of FEM – Variational Formulation of B.V.P. – Ritz Method – Finite Element Modelling – Element Equations – Linear and Higher order Shape functions – Bar, Beam Elements – Applications to Heat Transfer problems.	
UNIT II FINITE ELEMENT ANALYSIS OF TWO DIMENSIONAL PROBLEMS	**No. of Periods: 9+3**
Basic Boundary Value Problems in two-dimensions – Linear and higher order Triangular, quadrilateral elements – Poisson's and Laplace's Equation – Weak Formulation – Element Matrices and Vectors – Application to scalar variable problems - Introduction to Theory of Elasticity – Plane Stress – Plane Strain and Axi symmetric Formulation – Principle of virtual work– Element matrices using energy approach	
UNIT III ISO-PARAMETRIC FORMULATION	**No. of Periods: 9+3**
Natural Co-ordinate Systems – Lagrangian Interpolation Polynomials – Iso parametric Elements– Formulation – Shape functions -one dimensional , two dimensional triangular and quadrilateral elements -Serendipity elements- Jacobian transformation - Numerical Integration – Gauss quadrature – one, two and three point integration	
UNIT IV EIGEN VALUE PROBLEMS	**No. of Periods: 9+3**
Dynamic Analysis – Equations of Motion – Consistent and lumped mass matrices – Free Vibration analysis – Natural frequencies of Longitudinal, Transverse and torsional vibration – Solution of Eigen value problems - Introduction to transient field problems	
UNIT V NON-LINEAR ANALYSIS	**No. of Periods: 9+3**
Introduction to Non-linear problems - some solution techniques- computational procedure material non-linearity-Plasticity and visco-plasticity, stress stiffening, contact interfaces- problems of gaps and contact - geometric non-linearity - modeling considerations - Free and Mapped meshing -Mesh quality- Error estimate.	

Objective:

Introduction to Non-linear problems - some solution techniques- computational procedure material non-linearity-Plasticity and visco-plasticity, stress stiffening, contact interfaces- problems of gaps and contact - geometric non-linearity - modeling considerations - Free and Mapped meshing –Mesh quality- Error estimate.

Text Book:

T1. 1. Bathe K.J., "Finite Element Procedures in Engineering Analysis", Prentice Hall, 1990.
T2. David Hutton, "Fundamentals of Finite Element Analysis", Tata McGrawHill, 2005.

Reference Book:

1. Rao, S.S., "The Finite Element Method in Engineering", 6th Edition, Butterworth-Heinemann,2018.
2. Reddy,J.N. "Introduction to the Finite Element Method", 4 thEdition, Tata McGraw Hill, 2018.
3. Seshu.P, "Text Book of Finite Element Analysis", PHI Learning Pvt. Ltd., New Delhi, 2012.
4. Tirupathi R. Chandrupatla and Ashok D. Belegundu, "Introduction to Finite Elements in Engineering", International Edition, Pearson Education Limited, 2014..

UNIT I FINITE ELEMENT ANALYSIS OF ONE DIMENSIONAL PROBLEMS

PART A

1. **What is Finite Element Method (FEM)?**
   FEM is a numerical method used to solve boundary value problems by dividing the domain into smaller elements.
2. **What are weighted residual methods?**
   Methods that minimize the residual error by multiplying with weight functions and integrating over the domain.
3. **Define Ritz method.**
   A variational method where trial functions are used to minimize total potential energy.
4. **What is a shape function?**
   A function used to interpolate unknown values within an element.
5. **What is a bar element?**
   A one-dimensional element used to model axial deformation.
6. **What is variational formulation?**
   Converting differential equations into an equivalent energy minimization problem.
7. **Difference between linear and higher-order elements?**
   Linear elements have straight interpolation; higher-order elements have curved interpolation.
8. **What is element stiffness matrix?**
   A matrix relating nodal displacements to forces.
9. **What is boundary condition?**
   Constraints applied to the problem domain.
10. **Application of FEM in heat transfer?**
    Used to solve conduction problems in solids.

PART B

1. Explain the **historical development and basic concept of FEM** with advantages.
2. Derive the **stiffness matrix for a one-dimensional bar element**.
3. Explain **weighted residual methods** and compare different types.
4. Derive the **variational formulation for a boundary value problem**.
5. Explain the **Ritz method** with a suitable example.
6. Discuss **finite element modeling procedure step-by-step**.
7. Derive **shape functions for linear and quadratic elements**.
8. Formulate **element equations for beam elements**.
9. Solve a **heat transfer problem using FEM**.
10. Explain **assembly procedure and application of boundary conditions**.

UNIT II FINITE ELEMENT ANALYSIS OF TWO DIMENSIONAL PROBLEMS

PART A

1. **What is a 2D boundary value problem?**
   A problem defined over an area with boundary conditions.
2. **Define Poisson's equation.**
   $\nabla^2\varphi = f$
3. **Define Laplace equation.**
   $\nabla^2\varphi = 0$
4. **What is plane stress?**
   Stress condition where thickness stress is negligible.
5. **What is plane strain?**
   Strain in thickness direction is zero.
6. **What is weak formulation?**
   Integral form of governing equations.
7. **What is a triangular element?**
   A 2D element with three nodes.
8. **What is quadrilateral element?**
   A 4-node 2D element.
9. **What is elasticity theory?**
   Study of deformation under applied forces.
10. **What is principle of virtual work?**
    External work equals internal strain energy.

PART B

1. Derive the **stiffness matrix for a triangular element**.
2. Explain **Poisson's and Laplace's equations** in FEM formulation.
3. Derive the **weak form of a 2D boundary value problem**.
4. Explain **plane stress and plane strain conditions with equations**.
5. Derive **element equations using principle of virtual work**.
6. Explain **quadrilateral elements and their shape functions**.
7. Formulate FEM solution for a **2D scalar field problem**.
8. Discuss **elasticity theory and stress-strain relations**.

9. Derive **element matrices using energy approach**.
10. Solve a **2D heat conduction problem using FEM**.

UNIT III ISO-PARAMETRIC FORMULATION

PART A

1. **What is natural coordinate system?**
   A coordinate system (ξ, η) used in FEM.
2. **What is iso-parametric element?**
   Element where geometry and field variables use same shape functions.
3. **Define Jacobian.**
   Matrix relating natural and global coordinates.
4. **What is Gauss quadrature?**
   Numerical integration method.
5. **What is Lagrangian interpolation?**
   Polynomial interpolation using nodal values.
6. **What are serendipity elements?**
   Elements with fewer nodes than complete polynomial elements.
7. **What is numerical integration?**
   Approximate integration method.
8. **What is shape function?**
   Interpolation function.
9. **What is mapping?**
   Transformation between coordinate systems.
10. **What is 2-point Gauss quadrature?**
    Integration using two sampling points

PART B

1. Explain **iso-parametric formulation concept and advantages**.
2. Derive **shape functions using Lagrangian interpolation polynomials**.
3. Explain **natural coordinate system and mapping**.
4. Derive and explain **Jacobian transformation**.
5. Explain **numerical integration using Gauss quadrature**.
6. Compare **one-point, two-point, and three-point integration methods**.
7. Derive **shape functions for quadrilateral elements**.
8. Explain **triangular iso-parametric elements formulation**.
9. Discuss **serendipity elements and their formulation**.
10. Solve a **2D iso-parametric element problem step-by-step**.

UNIT IV EIGEN VALUE PROBLEMS

PART A

1. **What is dynamic analysis?**
   Study of structures under time-dependent loads.
2. **Define eigenvalue problem.**
   Problem to find natural frequencies and mode shapes.
3. **What is natural frequency?**
   Frequency at which system vibrates freely.
4. **What is lumped mass matrix?**
   Mass concentrated at nodes.
5. **What is consistent mass matrix?**
   Mass distributed over the element.
6. **What is free vibration?**
   Vibration without external force.
7. **Equation of motion?**
   $M\ddot{x} + Kx = 0$
8. **What is mode shape?**
   Deformation pattern in vibration.
9. **What is transient problem?**
   Time-dependent problem.
10. **What is damping?**
    Energy dissipation mechanism.

PART B

1. Derive the **equations of motion in FEM**.
2. Explain **free vibration analysis using FEM**.
3. Compare **consistent and lumped mass matrices**.
4. Derive **natural frequencies for bar and beam elements**.
5. Explain **eigenvalue extraction methods**.
6. Solve a **longitudinal vibration problem using FEM**.
7. Explain **transverse vibration of beams** using FEM.
8. Discuss **torsional vibration and its formulation**.
9. Explain **mode shapes and their physical significance**.
10. Discuss **transient dynamic problems and solution techniques**.

UNIT V NON-LINEAR ANALYSIS

PART A

1. **What is non-linear analysis?**
   Analysis where response is not proportional to load.
2. **Types of non-linearity?**
   Material, geometric, boundary.
3. **What is plasticity?**
   Permanent deformation after yielding.
4. **What is visco-plasticity?**
   Time-dependent plastic deformation.

5. **What is stress stiffening?**
   Stiffness change due to stress.
6. **What is contact problem?**
   Interaction between surfaces.
7. **What is mesh quality?**
   Measure of element accuracy.
8. **What is error estimation?**
   Estimating numerical solution accuracy.
9. **What is geometric non-linearity?**
   Large deformation effects.
10. **What is meshing?**
    Dividing domain into elements.

PART B

1. Explain **types of non-linearities (material, geometric, boundary)**.
2. Discuss **solution techniques for non-linear FEM (Newton-Raphson, etc.)**.
3. Explain **plasticity models and stress-strain behavior**.
4. Discuss **visco-plasticity and time-dependent effects**.
5. Explain **geometric non-linearity and large deformation analysis**.
6. Discuss **contact problems and gap elements**.
7. Explain **stress stiffening and its effects**.
8. Discuss **mesh generation techniques (free and mapped meshing)**.
9. Explain **mesh quality criteria and error estimation methods**.
10. Discuss **modeling considerations in non-linear FEM analysis**.